Rigid Body Mode Pointing Accuracy and Stability Criteria for an Orbiting Spacecraft

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Nomenclature

e_s, e_g = star tracker, rate gyro noise, rms
f_s, f_g = star tracker, rate gyro bandwidth
g = gain margin
I = spacecraft moment of inertia
$K_I, K_\theta, K_{\dot\theta}$ = integral, angle, rate gain
N_s, N_g = star tracker noise, rate gyro noise power spectral density
s = Laplace transform parameter
T_{gg} = peak gravity gradient torque
t_{gg} = gravity gradient torque
t_s, t_g = star tracker, rate gyro time constant
t_w, t_t, t_c = reaction wheel, tachometer, control moment gyro time constant
t_1 = noise filter time constant
ζ = CMG vibration mount damping ratio
ζ_c = compensation damping ratio
$\theta_g, \theta_s, \theta_{gg}$ = root mean square pointing error caused by rate gyro noise, star tracker noise, gravity gradient torque disturbances
θ_t = spacecraft angular deviation, rms
ω_c, ω_1 = compensation parameters
ω_{gg} = gravity gradient torque disturbance frequency
ω_n = vibration mount resonance frequency
ω_s = spacecraft bandwidth

Introduction

IN a Large Space Telescope (LST) spacecraft, fine pointing accuracy is one of the most critical requirements for astronomical study. In an ideal spacecraft attitude control system, a star tracker mounted on the spacecraft determines the line of sight to a guide star for comparison with a reference line on the spacecraft. When any deviation occurs because of spacecraft disturbance, an error signal from the star tracker is applied to the compensation network, which activates the CMG and reaction wheel to provide torque in the appropriate direction, thereby reducing the angular error from the star's line of sight. The principal source of disturbances can be classified as 1) disturbing torques which include gravity gradient, aerodynamics, solar radiation, magnetic, internal motion of components, and fuel or gas venting, and 2) sensor noise such as star tracker and gyro inherent random noise.

The effect of gravity gradient torque disturbances (which is an order of magnitude greater than the other torque disturbances and is sinusoidal with a frequency of two cycles per orbit) on pointing accuracy can be predicted very precisely. However, since the sensor noise, such as star tracker and gyro inherent noise, is random, their effect can only be predicted statistically. The most common method used is computer simulation with appropriate sensor model and adjusting spacecraft control laws for minimum pointing error, as presented by Harris.[1] The general investigation outlined in this Note theoretically relates spacecraft controls to pointing accuracy in terms of the sensor noise power spectrum. The results are presented in graphical form.

Derivation of the Governing Equations

The most commonly used simplified spacecraft attitude control system is investigated on a single-axis basis to parametrically relate pointing stability errors to system bandwidth for the three major disturbance sources, i.e., gravity gradient torques, gyro noise, and star sensor noise. For preliminary design, star tracker and gyro noise is assumed white Gaussian over the frequency range of interest, but results can be modified to the colored noise spectrum. Figure 1 is a basic system block diagram.

Based on the assumption that all the disturbances are statistically independent, the rms resulting from each disturbance is investigated independently. The actuator mechanization developed,[3] in which the control moment gyro shares torque with the reaction wheel, behaves almost as a linear torque device with a bandwidth much larger than the spacecraft's bandwidth. Neglecting the effect of sensor saturation, the transfer functions corresponding to each of three different disturbances are given as follows:

$$\frac{\theta_{gg}(s)}{t_{gg}(s)} = \frac{1/Is^2}{1+1/Is^2(K_I/s+K_\theta+K_{\dot\theta}s)} \qquad (1)$$

$$\frac{\theta_g(s)}{e_g(s)} = \frac{-(K_{\dot\theta}/Is^2)}{1+1/Is^2(K_I/s+K_\theta+K_{\dot\theta}s)} \qquad (2)$$

$$\frac{\theta_s(s)}{e_s(s)} = \frac{-(K_I/s+K_\theta)/Is^2}{1+1/Is^2(K_I/s+K_\theta+K_{\dot\theta}s)} \qquad (3)$$

The open-loop gain is given as

$$G(s)H(s) = 1/Is^2(K_I/s+K_\theta+K_{\dot\theta}s)$$

or

$$G(s)H(s) = K_I/Is^3(s^2/\omega_c^2+2\zeta_c s/\omega_c+1)$$

where

$$\omega_c = (K_I/K_{\dot\theta})^{1/2}, \quad \zeta_c = \tfrac{1}{2}[K_\theta/(K_I K_{\dot\theta})^{1/2}] \qquad (4)$$

For the control-loop configuration, the open-loop gain is the gain margin g at break frequency ω_c; i.e.,

$$g = |G(s)H(s)|_{s=j\omega_c}$$
$$g = 2\zeta_c K_I/I\omega_c^3 \qquad (5)$$

Also for a large ζ_c,

$$\omega_s = g\omega_c \qquad (6)$$

where ω_s is the crossover frequency. If a sufficiently large gain margin corresponding to a very low response peak is selected, the crossover frequency is approximately equal to the closed-loop bandwidth. Knowing the closed-loop bandwidth, the control laws can be conveniently expressed in terms of bandwidth with prescribed stability criterion.

From Eqs. (3–5), spacecraft control laws K_I, K_θ, and $K_{\dot\theta}$ can be expressed as

$$K_I = \left(\frac{4\pi^3 I}{\zeta_c g^2}\right)f_s^3, \quad K_\theta = \left(\frac{4\pi^2 I}{g}\right)f_s^2, \quad \text{and} \quad K_{\dot\theta} = \left(\frac{\pi I}{\zeta_c}\right)f_s \qquad (7)$$

Effect of Gravity Gradient Torque Disturbance on Pointing Error

The angular error caused by the gravity gradient torque disturbance can be expressed as a function of the system bandwidth. Rewriting Eq. (1)

$$\frac{\theta_{gg}(s)}{t_{gg}(s)} = \frac{s}{Is^3+K_I(s^2/\omega_c+2\zeta_c s/\omega_c+1)}$$

The magnitude of the response per unit gravity gradient torque can be evaluated as

$$\left|\frac{\theta_{gg}}{t_{gg}}\right|_{s=j\omega_{gg}} = \left|\frac{j\omega_{gg}}{-j\omega_{gg}{}^3 I+K_I[-(\omega_{gg}/\omega_c)^2+2\zeta_c j(\omega_{gg}/\omega_c)+1]}\right|$$

Using the value of K_I and ω_s and simplifying

$$\left|\frac{\theta_{gg}}{t_{gg}}\right|_{s=j\omega_{gg}} = \left|\frac{j\omega_{gg}}{K_I(\omega_{gg}/\omega_c)^2[-1+2j\zeta_c(\omega_c/\omega_{gg})+(\omega_c/\omega_{gg})^2]}\right| \quad \text{for} \quad \omega_{gg} \ll 1$$

The steady-state pointing error per unit gravity gradient torque can be expressed as a function of system bandwidth by introdu-

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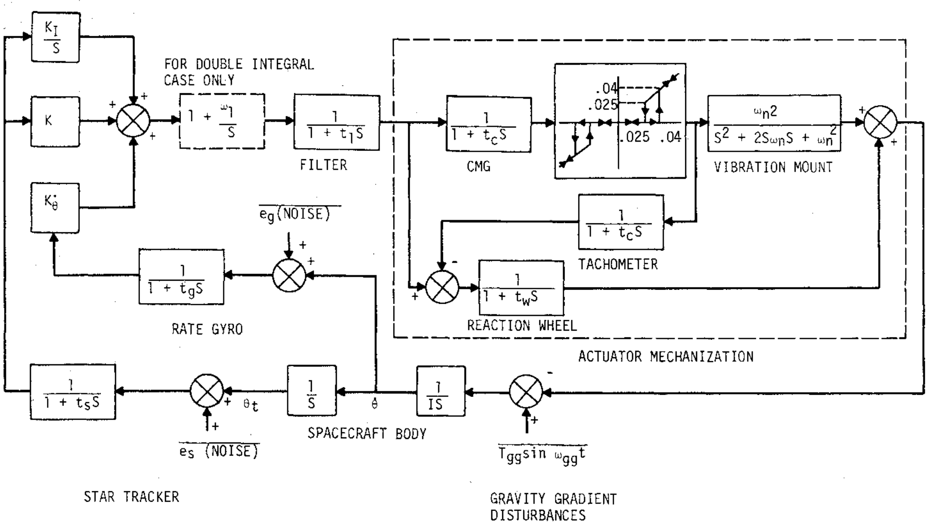

Fig. 1 Basic system block diagram for proportional, derivative, and integral or double integral control.

cing a new parameter, $s = j\omega_s = jg\omega_c$, into the previous equation, as follows:

$$\left|\frac{\theta_{gg}}{t_{gg}}\right| = \left|\frac{2\zeta_c}{I\omega_{gg}s[s^2/g^2\omega_{gg}^2 - 2\zeta_c s/g\omega_{gg} + 1]}\right| \tag{8}$$

Note that ω_s is system bandwidth, not a natural frequency. This transfer function, having poles in the right half-plane, does not represent a physical system. It is only a convenient method of plotting the pointing error using the Bode diagram for magnitude as a function of frequency.

Rate Gyro Noise Effect on Pointing Accuracy

The transfer function describing the gyro noise effect on pointing accuracy is derived in Eq. (2) using Eqs. (4) and (5) and simplifying

$$\frac{\theta_g(s)}{e_g(s)} = -\frac{s}{2\zeta_c s^3/g\omega_c + s^2 + 2\zeta_c\omega_c s + \omega_c^2}$$

Using the Phillips-Weiner method for evaluating the mean square,[2] the mean-square error is

$$\theta_g^2 = \frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{N_g}{2}\frac{\theta_g(s)}{e_g(s)}\frac{\theta_g(-s)}{e_g(-s)}ds$$

where N_g is the spectral density of rate gyro noise, which is equal to e_g^2/f_g in (arc sec/sec)^2/Hz because the gyro noise power spectral density is considered to be constant over f_g. For $f_g \gg f_s$

$$\theta_g^2 = \frac{N_g}{4\pi j}\int_{-\infty}^{\infty}\frac{\theta_g(s)}{e_g(s)}\frac{\theta_g(-s)}{e_g(-s)}ds$$

The complex integration can be carried out with the aid of the tables shown in Ref. 3. Carrying out the integration and simplifying

$$\theta_g = [N_g g^2/16\pi(g-1)\zeta_c]^{1/2}(1/f_s)^{1/2} \tag{9}$$

Star Tracker Noise Effect on Pointing Error

With appropriate manipulations using Eqs. (3–6), the transfer function describing the star tracker noise effect is

$$\frac{\theta_s(s)}{e_s(s)} = \frac{[1 + (2\zeta_c/\omega_c)s]}{(2\zeta_c/g\omega_c^3)s^3 + s^2/\omega_c^2 + 2\zeta_c/\omega_c s + 1}$$

The mean-square error is given by

$$\theta_s^2 = \frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{N_s}{2}\frac{\theta_s(s)}{e_s(s)}\frac{\theta(-s)}{e(-s)}ds$$

where N_s is the power spectral density of the star tracker noise, i.e., $N_s = e_s^2/f_{st}$ in (arc sec/sec)^2/Hz. Since noise is considered white, N_s is constant over f_{st}. For $f_{st} \gg f_s$

$$\theta_s^2 = \frac{N_s}{4\pi j}\int_{-\infty}^{\infty}\frac{\theta_s(s)\theta_s(-s)}{e_s(s)e_s(-s)}ds$$

Carrying out the integration and simplifying,

$$\theta_s = \left[\frac{\pi(4\zeta_c^2+1)N_s}{4\zeta_c(g-1)}\right]^{1/2} f_s^{1/2}$$

Since all of the disturbances considered are statistically independent, the total rms error has the following form:

$$\theta_t = [(\theta_{gg})^2 + (\theta_g)^2 + (\theta_s)^2]^{1/2} \tag{10}$$

Setting the derivative of θ_t with respect to f_s, $\partial\theta_t/\partial f_s$, equal to zero gives the expression from which the optimum bandwidth for minimum pointing error, which defines the control laws according to Eq. (7), can be solved. As can be seen from Eq. (8), the analytical solution is cumbersome; therefore, a graphical approach is followed. All of the three terms in Eq. (8) are plotted individually in Fig. 2 for the following system constants: $\omega_{gg} = 0.002$ rad/sec, $T_{gg} = 0.15$ ft/lb, $I = 90{,}000$ slug/ft^2, $g = 4$ (or 12 db), and $\zeta_c = 0.707$.

Experimental Results

Two independent random noise signal generators were used to represent star tracker and rate gyro noise for a complete system analog computer simulation. The actuator mechanization,[3] in which the control moment gyro is modeled as a nonlinear device that shares torque with the reaction wheel in the deadband region, is used to evaluate over-all system performance. With appropriate scaling, spacecraft bandwidth was varied for a fixed stability criterion.

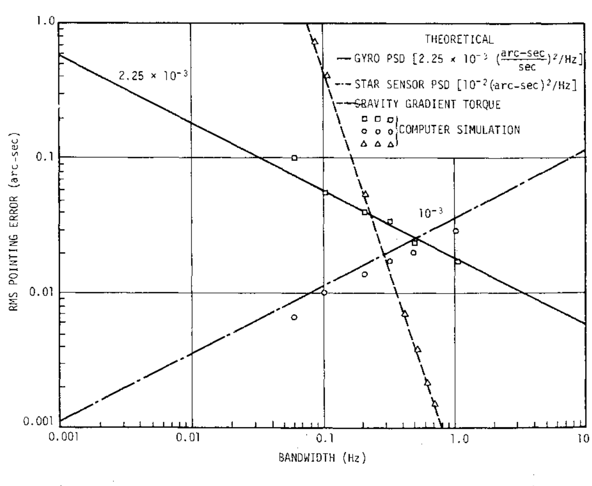

Fig. 2 LST pointing errors caused by sensor noise and gravity torque disturbances—proportional, derivative, and integral control.

Discussion of Results

As a result of dynamic analysis, a good agreement between the results of the theoretical analysis and the computer simulation exists, as shown in Fig. 2. The adequacy of the investigation outlined for preliminary design purposes attests to the system synthesis.

The double integral case[3] shown in Fig. 1 was investigated on a similar basis. Since the gyro noise generally is not white, Truncale, Koenigsberg, and Harris[4] measured the noise power spectrum for different kinds of existing gyros. Expressing the measured data in appropriate theoretical form and calculating the pointing error as a function of bandwidth, a compromise can be made to select the gyro required for a given star tracker. With reference to the results shown, the solutions are presented only to indicate the general characteristics, i.e., mainly the pointing error for a fixed stability criterion. A control system design for a given sensor would most likely have different characteristics because of other systems' nonlinearities. However, for preliminary design purposes, a general statement pertaining to the LST[5] fine pointing error may be made for general-purpose system synthesis, and the relations describing the governing equations can be modified to other kinds of orbiting spacecraft.

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Time-Optimal Pitch Control of Satellites Using Solar Radiation Pressure

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Nomenclature§

A_i	= control plate area, $i = 1, 2$
C, C_i	= solar parameter, $i = 1, 2$
C^*	$= [2/3(3)^{1/2}]C - \lvert 3K_i \sin\psi_e \cos\psi_e \rvert$
I_x, I_y, I_z	= principal moments of inertia of the satellite
K_i	= inertia parameter, $(I_z - I_y)/I_x$
P	= pericenter
Q_ψ	= generalized force due to solar radiation pressure
R_p	= distance between the pericenter and the center of force
i	= inclination of the orbital plane with the ecliptic
n	$= (3K_i \cos 2\psi_e)^{1/2}$, $-\pi/4 < \psi_e < \pi/4$
$\bar{u}$	= unit vector in the direction of the sun
$\mathbf{u}(\theta)$	= control vector
p_0	= solar radiation pressure
x, y, z	= principal body coordinates
x', y', z'	= inertial coordinates
x_0, y_0, z_0	= rotating coordinate system with x_0 normal to the orbital plane and y_0 along the local vertical
δ_i	= plate rotation, $i = 1, 2$
ε_i	= distance between the center of pressure of control plate and satellite center of mass, $i = 1, 2$
θ	= orbital angle
θ_s, θ_f	= switching time and final time, respectively
μ	= gravitational constant
ζ	$= \omega + \theta + \psi - \tan^{-1}(\tan\phi \cos i)$
ρ, τ	= reflectivity and transmissibility of plate surfaces, respectively
ϕ	= solar aspect angle
ψ, ψ_e	= pitch attitude and its nominal orientation, respectively
ω	= angle between the line of apsides and the line of nodes

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§ Primes indicate differentiation with respect to the orbital angle θ.

Introduction

THE use of solar radiation pressure for attitude stabilization of satellites has been a subject of considerable discussion.[1-4] Its ability to provide libration damping and general three-axis attitude control to gravity-oriented as well as spinning vehicles has been clearly established.[5,6] An experiment aboard Mariner IV spacecraft, where solar pressure was used in conjunction with active gyros, demonstrated the effectiveness of the controller in practice.[7] The feasibility of the concept being well established, the next logical step would be to direct efforts at improving the performance of the system through the implementation of optimal control laws. As the energy required to operate the solar control plates is quite small and could be generated easily through the use of solar cells, the performance index need only include the damping time which is of prime concern.

Time-optimal control of multidegree-of-freedom systems, such as the coupled roll-yaw-pitch motions of a satellite, generally involves software complexity as the solution of a two-point boundary-value problem is required. On the other hand, a single-degree-of-freedom system may lend itself to an analytical synthesis of the time-optimal switching criterion. This is significant since if successful, it not only could be applied to several situations of practical importance (platform pitch control of a spinning satellite, e.g., the proposed magnetic-solar hydrid control system[8] or pure pitch control of a gravity gradient system as in the case of COSMOS-149[9]) but may also suggest switching

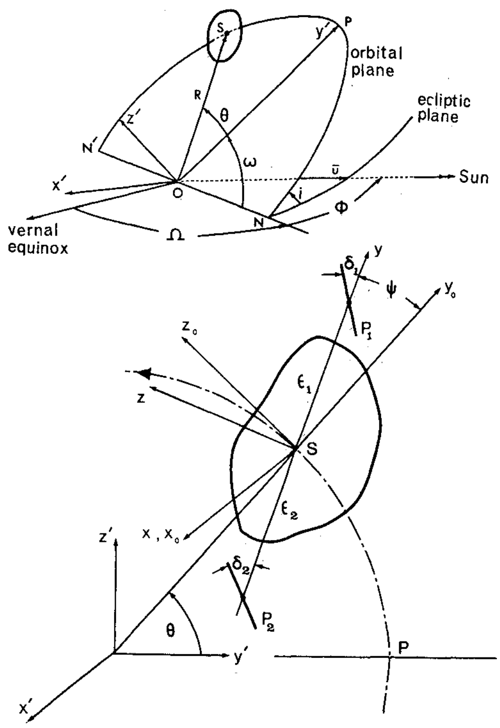

Fig. 1 Geometry of motion.